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Adaptive Friction Compensation or Motion Stabilization

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The motion stabilization problem for the systems with friction force acting as disturbance has been considered. Two friction observer algorithms have been proposed to estimate the Coulomb friction force. One estimates the magnitude of the friction force only, while the other estimates the state and the friction force, simultaneously. The latter is robust to the measurement noise but needs more computation. The simulation study has showed that the compensation using the proposed friction observers greatly improves the stabilization performance.

Key Words: Motion Stabilization, Friction Observer, Feedforward Friction Compensation, Stabilization Performance

1. Introduction

In many practical situations, the friction force can be considered as a constant disturbance. Hence, the integral action is included in the controller to overcome its effect such as steady state error. However, when friction changes in both its magnitude and direction, especially in its direction, it cannot be assumed as a constant disturbance. In Fig. 1, the directional change of friction in contacting surfaces gives rise to the degradation of stabilization performance, that is, the mass is disturbed by the friction force so that its velocity cannot be made to be zero. In such circumstances, conventional methods using an integrator would not meet performance specifications. It is referred to as 'motion stabilization problem' for convenience. Motion stabilization problem has been frequently appeared in the field of advanced weapon control such as the gun-turret systems and the stabilized gun-mirror system of armored vehicles (Lin, 1994).

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Many studies have been carried out to reduce the effect of friction by both control and tribology researchers (Haessig and Friedland, 1991 ; Tung et. al, 1993 ; Armstrong-Helouvry, 1993 ; Yang, 1992 ; Friedland and Park, 1992 ; Yang and Chu, 1993). Among them, Friedland and Park (1992) studied on the adaptive friction estimation and compensation for the single frictional system. They suggested a nonlinear observer algorithm for estimating a friction force. However, their method should be modified for the motion stabilization applications due to the base motion as shown in Fig. 1.

In this note, two friction observer algorithms are proposed for the system shown in Fig. 1. One

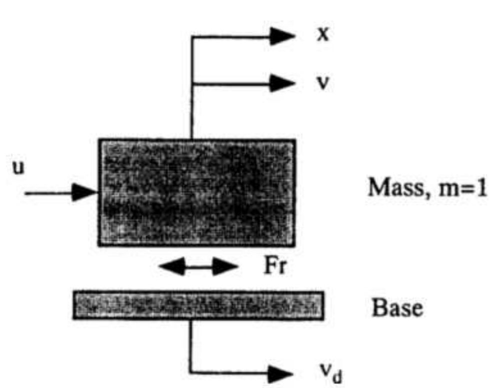


Fig. 1 Simplified motion stabilization problem. The motion of the base is transmitted to the mass due to the friction force

is derived through the minor change of Friedland and Park's approach. The other is developed as a state and friction observer for the noisy environment. The effects of the friction compensation are discussed qualitatively through the order analysis. It is shown that the proposed algorithm greatly improves the stabilization performance by a simulation.

2. System Descriptions

Friction is one of the major nonlinear elements which can degrade the motion stabilization performance in the control systems. Figure 1 shows a simplified motion stabilization problem. Disturbance is transmitted to the mass through the friction mechanism in the contact surfaces between the base and the mass. Figure 2 represents the extended block diagram including a feedforward friction compensator to reduce the effect of the friction. The transfer function $P(s)$ denotes the prefilter which converts the reference signal into the allowable signal. If there is no friction compensator, good stabilization performance may not be obtained because of the friction effects. The system shown in Fig. 1 is governed by the equation

$$\dot{v} = u - Fr(v_r) \tag{1}$$

where v and $v_r = v - v_d$ are the absolute velocity of the mass and the relative velocity in the contact surfaces, respectively, and u is the control input force. The scope of the friction treated in this note is restricted to the memoryless friction which depends on the relative velocity only. This type of friction force (Armstrong-Helouvry, 1991) is frequently described by

$$Fr(v_r) = (fr + c_o \mid v_r \mid + c_1 e^{c_2 \mid v_r \mid}) s(v_r) \tag{2}$$

where fr and c_i are constants or slowly time varying and $s(\cdot)$ denotes the signum function. For convenience, Eq. (2) can be rewritten as follows :

$$Fr(v_r) = (f_r + \Delta fr) s(v_r) \tag{3}$$

where $\Delta fr = c_o \mid v_r \mid + c_1 e^{c_2 \mid v_r \mid}$. Generally, it has been assumed that the Coulomb friction, fr is dominant in most cases. Hence, in this note, only the Coulomb friction is treated for the estimation and compensation. However, the ignored part of friction, Δfr is included in the analysis of the compensator.

3. Friction Observer Algorithms

To construct a friction observer, the following assumption is necessary.

Assumption 1 :

(i) The Coulomb friction, fr is dominant and its magnitude is slowly time varying.

(ii) The information such as u, v and $s(v_r)$ is available.

The following theorem presents a friction observer algorithm.

Theorem 1 : Under the assumption 1, the Coulomb friction force in Eq. (1) is estimated by the following asymptotic observer :

$$\dot{z} = k\, s(v_r)\{(u - \hat{f}r\, s(v_r))\} + k\, v \frac{d}{dt} s(v_r) \tag{4}$$

$$\hat{f}r = z - k\, v\, s(v_r) \tag{5}$$

where $k > 0$ and z and $\hat{f}r$ are the state variable of friction observer and the estimated friction force, respectively. Or equivalently,

$$\dot{\hat{f}}r = -k\hat{f}r + ks(v_r)\{(u - \dot{v}\}. \tag{6}$$

(Proof) To see the behavior of the proposed observer, the error dynamics must be derived. Let $e_{fr} = fr - \hat{f}r$ be the estimation error. Then, the error dynamics is

$$\begin{aligned} \dot{e}_{fr} &= \dot{f}r - \dot{\hat{f}}r \\ &\cong -\dot{\hat{f}}r \\ &= -\dot{z} + k\dot{v}s(v_r) + kv\frac{d}{dt}s(v_r) \\ &= -k(fr - \hat{f}r + \Delta fr)s(v_r)s(v_r) \\ &= -k(e_{fr} - \Delta fr) && (7) \\ &\cong -ke_{fr} && (8) \end{aligned}$$

Therefore, the error dynamics is asymptotically stable as long as k is positive. Eq. (6) can be obtained by eliminating z in Eqs. (4) and (5).

(Q. E. D)

Note that the observer is a modified version of the result proposed by Friedland and Park (1992). Even though the dynamics of the given observer in Eqs. (4) and (5) is nonlinear, the estimation error dynamics is linear as can be seen in Eq. (8). Hence, the estimation performance can be easily understood without any simulation.

It is necessary to differentiate $s(v_r)$ or v to construct the friction observer. The exact differentiation of $s(v_r)$ with respect to time is impossible because it goes to infinity at sign changing instances, but its approximated value such that

$$\frac{d}{dt}s(v_r)\cong\frac{v_r(t)-v_r(t-T)}{T} \tag{9}$$

for sufficiently small T, may work well because v, which is multiplied by $\frac{d}{dt}s(v_r)$ in Eq. (4), would be small if the regulation is successful. However, it is clear that the usage of $\frac{d}{dt}s(v_r)$ is a difficult problem in the numerical calculation. Due to this problem, Eq. (6) seems to be more realizable than the form of Eqs. (4) and (5). The availability for the first order differentiation of the measurements has been generally assumed for constructing the unknown input observers. However, it leads to the noisy estimation problem. One can see that the measurement noise is amplified by the observer gain, k and directly included in the estimate of fr. Hence, this observer is sensitive to noise and the only way to reduce the effect of noise is to reduce the gain, k as small as possible. Note that it also makes the observer dynamics slow. If the friction magnitude is slowly time varying and the measurement is noisy, it is appropriate that one sets the initial condition of friction observer state to the steady state value obtained by some trial runnings of friction observer and tunes the gain k as small as possible.

The following theorem gives the friction observer which can avoid the aforementioned problems.

Theorem 2 : Under the assumption 1, the Coulomb friction force in Eq. (1) is estimated by the following asymptotic observer :

$$\dot{\hat{v}}=u-\hat{f}rs(v_r)+k_1(v-\hat{v}) \tag{10}$$
$$\dot{\hat{f}}r=-k_2s(v_r)(v-\hat{v}) \tag{11}$$

where k_1 and k_2 are positive constant gains.

(Proof) Let's define an additional error such that $e_v=v-\hat{v}$. Then, the error dynamics is derived under the assumption 1 as follows :

$$\dot{e}_v=-k_1e_v-s(v_r)e_{fr} \tag{12}$$
$$\dot{e}_{fr}=k_2s(v_r)e_v \tag{13}$$

To show the convergency, it is necessary to choose the following Lyapunov function.

$$V(e_v,\ e_{fr})=k_2e_v^2+e_{fr}^2 \tag{14}$$

Then, its derivative with respect to time is

$$\dot{V}(e_v,\ e_{fr})=-2k_1k_2e_v^2<0 \tag{15}$$

for $k_1>0$ and $k_2>0$. (Q. E. D)

Note that Eq. (10) is the usual form of the state observer and Eq. (11) is drawn from Eq. (6) in theorem 1 by replacing $\hat{v}$ with as follows :

$$\dot{\hat{f}}r=-k\hat{f}r+ks(v_r)\{u-\dot{\hat{v}}\} \tag{16}$$

It is not required to know the derivatives of the signum function or v for realizing the friction observer. The friction observer is robust to the measurement noise because the measurement noise in v or $s(v_r)$ is filtered through the pure integratior to calculate the friction estimate in Eq. (11).

The error dynamics is piecewise linear as shown in Eqs. (12) and (13), and it has the characteristic equation such that

$$s^2+k_1s+k_2=0 \tag{17}$$

during the sign is kept unchanged. Hence, one can assign the observer gains based on the above characteristic equation. However, the proper simulation is necessary to confirm the behavior of the observer during the sign change.

For the two proposed friction observers, it can be easily proven that the accuracy of the estimated friction value is determined by the ignored friction components, Δfr. Detailed derivation is omitted for its simplicity. At the steady state, the order of the estimation error is

$$e_{fr}\sim O(\Delta fr) \tag{18}$$

4. Feedforward Friction Compensation

The total control input consists of two parts as can be seen in Fig. 2. One is from the conventional feedback controller and the other from the feedforward friction compensator as follows :

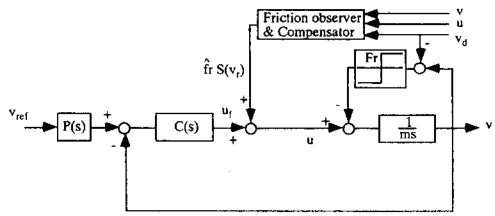


Fig. 2 Extended block diagram including a friction compensator for the motion stabilization

$$u = u_f + \hat{f}r s(v_r) \tag{19}$$

where u_f is the control input from the conventional feedback controller. When u in Eq. (19) is used, dynamics of the totally compensated system is expressed as follows:

$$\dot{v} = u_f - (e_{fr} + \Delta fr) s(v_r) \tag{20}$$

Since the order of e_{fr} is $O(\Delta fr)$ as in Eq. (18), the term, $e_{fr} + \Delta fr$ also has the same order of $O(\Delta fr)$. Therefore, Eq. (20) can be represented by

$$\dot{v} = u_r - c\Delta fr\ s(v_r) \tag{21}$$

where c is a proper scaling factor which may be time varying but cannot alter the order of $O(\Delta fr)$. Comparing Eq. (21) with Eq. (1), one can see that the input disturbance is greatly reduced and so that the remaining portion of the friction force is much more manageable by the feedback controller. However, the compensation scheme in Eq. (19) has some problems that need further investigation. One is due to the inaccuracy of the friction model and the other due to the measurement noise at the low relative velocity, which causes the wrong estimation of $s(v_r)$ and therefore, wrong direction of the compensatory input. It is well known that friction has memory in some applications (Walrath, 1984). In these cases, the compensation using Eq. (19) would lead to the bad control performance because the direct reversal of friction force is not occurred just after the sign change of the relative velocity. Hence, the complete compensation of friction at low velocity range is impossible in reality and the avoidance of the stick-slip cannot be guaranteed. However, several works have proposed the feedback controller design methods to achieve the steady sliding at low velocity without the additional friction compensators (Dupont, 1994). Therefore, it is necessary to combine the feedback controller design techniques and the proposed compensation method to obtain good control performance. It can be expected that design of the feedback controllers becomes easier, because the level of the input disturbance is small, as can be seen in Eq. (21), when the friction compensator is active.

5. Simulation

The velocity disturbance is generated by adding some harmonics so that the direction of the friction force changes frequently. The friction model used for the simulation is as follows:

$$Fr(v_r) = (0.05 + 0.01 \mid v_r \mid + 0.01 e^{-138 |v_r|}) s(v_r) \tag{22}$$

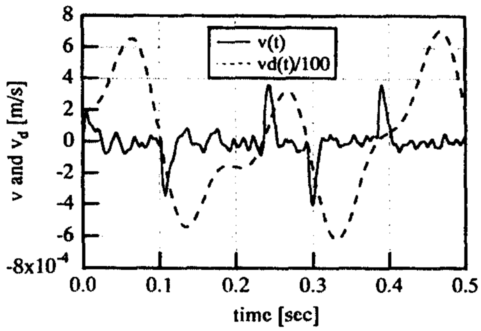
The prefilter $P(s)$ is set to 1 and a lag compensator with an integrator is used for the feedback controller $C(s)$ as follows:

$$C(s) = \frac{1.509 \times 10^5 (s + 103.6)}{s(s + 603.5)} \tag{23}$$

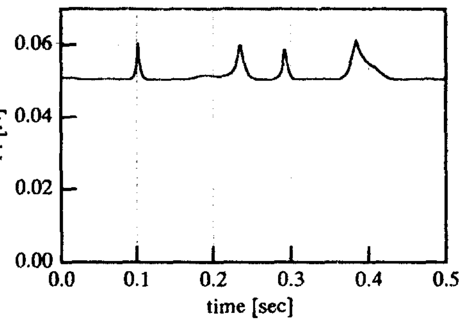
It is assumed that the measurement noise is included in the measurement of v as follows:

$$v_m = v + n \tag{24}$$

where v_m is the measured velocity. The S/N ratio is about 20 dB. For convenience, friction observer 1 and friction observer 2 represent those based on theorem 1 and theorem 2, respectively. Figure 3 shows the stabilizing performance obtained by using the feedback controller alone. At the zero relative velocity, the mass is destabilized. At those instances, not only the friction magnitude is increased due to the static friction phenomenon but also the direction of the friction is changed. Note that the destabilization effect is mainly caused by the directional change of the friction force rather than the magnitude change. In this operating circumstance, two friction observers are constructed to estimate the friction magnitude. The values of k, k_1 and k_2 are 20, 90 and 1800, respectively. Figure 4 shows the estimation performances in the noisy environment. Note that the observer gains are chosen for two observers to have the similar convergency time for the valid compari-

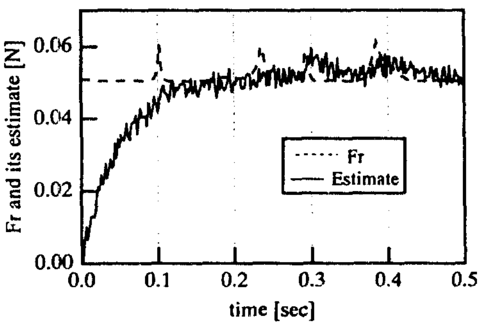


(a) Velocity disturbance and velocity of mass

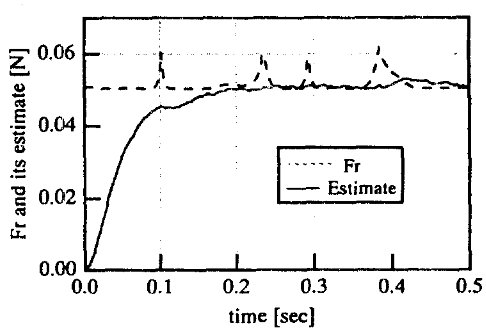


(b) Magnitude of real friction force

Fig. 3 Stabilization performance obtained by the feedback controller alone

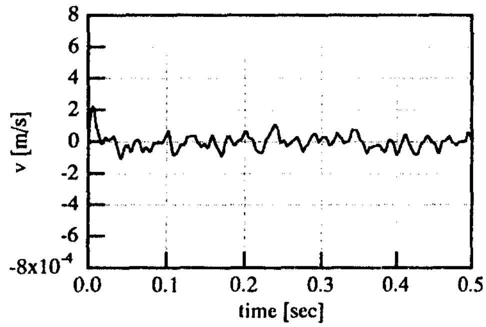


(a) Friction observer 1

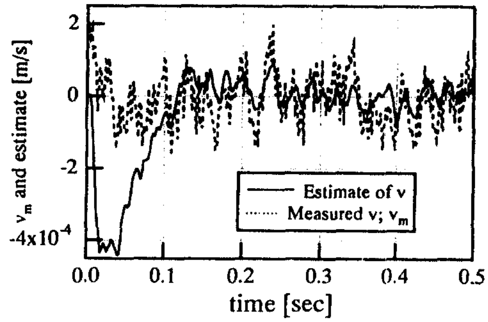


(b) Friction observer 2

Fig. 4 Estimation performances of two friction observers in the noisy environment



(a) Velocity of mass



(b) Measured velocity, v_m and its estimate

Fig. 5 Stabilization performance obtained by adding a friction compensator based on the friction observer 2

son. As stated in sec. 3, the friction observer 2 is robust to the measurement noise. Figure 5 shows the stabilization performance obtained by including the friction compensator based on the friction observer 2. Note that the filtered signal shown in Fig. 5(b) is obtained as a byproduct. Hence, one can use it instead of the measured signal, v_m for the feedback signal in the feedback controller. Figure 5(a) is the result of including the friction compensator and using $\hat{v}$ instead of v_m for the feedback signal. Compared with Fig. 3(a), it can be shown that the stabilization performance is greatly improved.

6. Conclusions

Friction compensation is necessary for the motion stabilization. Novel friction estimation algorithms have been proposed to construct friction compensators. One is a reduced order friction observer which is simple and the modified version of Friedland and Park's work. The other is a simultaneous state and friction observer which is robust to the measurement noise. The simulation study has showed that the friction compensator reduces the effective input disturbance through the feedforward compensation and, henceforth, improves the stabilization performance even though the exact friction compensation is impossible.

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